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Topology Control for Predictable Delay-Tolerant Networks based on Probability

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Abstract

In wireless networks, topology control can improve energy effectiveness and increase the communication capacity. In predictable delay tolerant networks (PDTNs), intermittent connectivity, network partitioning, and long delays make most of the researchers focus on routing protocol, and the research on topology control is in the very early stage. Most existing topology control approaches for PDTN assume the underlying connections are deterministic, which excludes a large amount of probabilistic connections existing in the challenged environments. In this paper, probabilistic connections are taking into consideration. The predictable delay tolerant networks are modeled as a three dimensional space-time weighted directed graph which includes spatial, temporal and connection probability information. The topology control problem is formulated as finding a sub graph to balance the energy cost and data transferring reliability. This problem is proved to be NP-complete, and two heuristic algorithms are proposed to solve the problem. The first one is to find a sub graph that assures the maximum connection probability between each pair of nodes. The other one is to find the sub graph in which the connection probabilities between each pair of nodes satisfy the given threshold with the minimum energy cost. Extensive simulation experiments demonstrate that the proposed topology control algorithms can achieve our goal.

Keywords: Predictable delay tolerant networks, Connection probability, Topology control

1. Introduction

In recent years, Delay/Disruption Tolerant Networks (DTNs) \cite{1} have attracted more and more researchers’ attention \cite{2}. Predictable Delay/Disruption Tolerant Networks (PDTNs) is a kind of DTNs whose topology is known a priori or can be predicted over time, such as pocket switched networks based on human mobility, vehicular networks based on public buses or taxi cabs, and space satellite networks.

Due to the characteristics such as intermittent connectivity, partitioned network, long delays and node mobility, the DTNs/PDTNs are not fully connected and the topology dynamically changes over time. How to transmit data between each pair of nodes successfully is a challenging problem, which makes many researchers focus on routing protocols \cite{3} - \cite{9}.

Topology control has been studied widely in wireless ad hoc and sensor networks, which can maintain network connectivity while reducing the energy cost and radio interference. In general, topology control can avoid extra resource consumption of data transmitting. It is also true in DTNs, especially PDTNs. Instead of trying to using all possible transmitting opportunities, topology control mechanism can select certain transmitting opportunities to delivery data in such network to satisfy the reliability requirement with lower resource consumption.

However, using the existing topology control methods to PDTNs directly is not feasible because the topology of PDTNs is time evolving and the routing paths intermittently exist. Some researches on routing protocol mentioned above considered dynamic topology and intermittent connectivity to make better routing decision to maximize the data delivery while maintain a reasonable resource cost.

Temporal characteristic is not considered in all these solutions. Huang et al. \cite{10} takes the temporal characteristic into consideration and model PDTNs as a directed space-time graph including spatial and temporal information. The topology control problem is formulated as finding a sub graph. However, they assume the underlying connections are deterministic.

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In practical wireless environments, transitional region phenomenon exists [11][12]. Beyond the "always connected" region, there is a transitional region where a pair of nodes may probabilistically connect. The links between such pairs of nodes is called lossy links. The lossy links provide probabilistic data transmission, which means the nodes connected by them are not fully connected but reachable. It has been reported that in the wireless network, especially DTN, the number of lossy links are much more than the number of fully connected links. It has been proved that utilizing lossy links appropriately can reduce the energy cost and improve the data delivery chance [13][18][19][21].

If the underlying links are deterministic, the connection between a pair of nodes either exists or not. However, considering lossy links will make the connection between a pair of nodes exists probabilistically. The quality of connection needs to be evaluated to get better transferring paths. The Quality of Connectivity (QoC) is used to evaluate the quality. It measures how easily and reliably a packet can be sent between a pair of nodes in a network. It complements the use of capacity to measure the quality of a network in saturated traffic scenarios and provides a native measure of the quality of end-to-end network connections. The QoC is calculated as the connection probability between a pair of nodes in the network. An example is shown Fig.1, obviously its value depends on the topology and underlying link qualities. In Fig.1, the number beside the direct edge represents the corresponding link probability, the connection probability of each pair of nodes can be calculated by inclusion-exclusion principle of probability theory. For example, the QoC indicating the quality of connection between V0 and V3 in case (a) is QoC$_{a} = 0.9*0.8*0.95=0.684$ since there is only a serial three hop connection. The connection probability is simply the product of underlying link probabilities. In case (b), there are two parallel connections between V1 and V3, either of these connections can be used to transfer, which leads to higher QoC value, QoC$_{b} = 0.9*0.8*0.95^2+(1-0.9)*0.95=0.8784$. In case (c), there are two parallel connections between V0 and V2. Similar with case (b), either of these connections can be used to transfer, which leads to higher QoC value, QoC$_{c} = (1-(1-0.9)^2)*0.95 = 0.8968$. Obviously, case (b) and (c) are better than (a) since multiple paths exist.

Our approach optimizes the tradeoff between the energy cost and connection quality by making use of lossy links. The goal is to ensure the connection probability of each pair of nodes is maximized or reach a certain threshold with the minimum energy cost. Our major contributions are summarized as follows:

(1) We model PDTNs as a three dimensional space-time weighted directed graph, which include lossy links. The topology control problem is defined as constructing a sub graph(minimum cost sparse structure) from the original three dimensional space-time graph such that: Maximizing connection probability of each pair of nodes, or satisfying the given connection probability threshold with lower energy cost. We also prove this problem is NP-complete.

(2) We propose two heuristic topology control algorithms. The first one is to find a sub graph that assures the maximum connection probability between each pair of nodes. The other one is to find the sub graph in which the connection probabilities between each pair of nodes satisfy the given threshold with the lower energy cost. Extensive simulation experiments demonstrate that the proposed algorithms can provide a certain data transferring reliability with lower energy cost.

The remainder of the paper is organized as follows. Section 2 reviews related work. In Section 3, we describe the three dimensional space-time weighted directed graph model and define the topology control problem. In Section 4, two heuristic weighted directed graph algorithms proposed in this paper are described. Section 5 discusses the simulation results. Finally, Section 6 concludes the paper and discusses the future directions.

2. Related work

Topology control has been studied widely in ad hoc and sensor networks. The aim of topology control is to maintain network connectivity with the least energy cost. Most topology control protocols can be classified into two categories: deterministic topology control
and probabilistic topology control. Deterministic topology control can also be classified into power-adjustment based method and Minimal Connected Dominating Set (MCDS) based method. MCDS based method can be further classified into clustering based method and geometrical structure based method.

Power adjustment based methods maintains the global network connectivity through adjusting the transmission power of each node to the minimal. This problem has been proved as NP-hard and researchers have proposed many heuristics algorithms to solve the problem [15]. MCDS removes unnecessary edges by seeking the Minimal Connected Dominating Set (MCDS). Geometrical structure-based method [14][16] constructs an sparse network by removing unnecessary links from the original graph based on location information. Clustering-based method [17] builds a hierarchical structure through forming cluster. The selected cluster heads serve as the skeleton nodes. The above topology control methods all repeatedly perform constructing algorithm to deal with topological changes. Since the constructing algorithm is distributed or local, cycling cost is smaller. However, this algorithm all assumes that the graph is completed connected and excludes the lossy links.

To the best of our knowledge, the research on topology control of delay tolerant network is in the very early stage. Most researches focus on data routing, which can be classified into three categories: opportunity-based routing, message-ferry-based routing and prediction-based routing. Opportunity based schemes [6][22] aim to deliver messages to the destination through randomly transmitting data hop by hop. There is no guarantee that data can deliver successfully. Multiple copies of the same message may be produced and flooded to make data deliver successfully as far as possible. However, it leads to more buffer space and more bandwidth consumption. Message-ferry-based methods [4][5][23] often use an additional mobile nodes as ferries to transmit messaging. These message ferries’ trajectory can be controlled, so the delivery performance will be greatly improved by store-and-carry. Of course, because of the need for additional mobile node, these nodes need extra consumption and load control. In prediction-based methods [7][9], routing protocols make use of some delivery estimation metrics to predict the relative of contacts with the successful delivery, such as delivery delay or probability and statistics of contact history.

Recently, Y. Zeng et al. [26] present a directional routing and scheduling scheme (DRSS) for green vehicle DTNs by using Nash Q-learning approach that can optimize the energy efficiency with the considerations of congestion, buffer and delay. A. Dvir, et al.[27] present an alternative, highly agile approach called backpressure routing for DTNs, in which routing and forwarding decisions are made on a per-packet basis. Using information about queue backlogs, random walk and data packet scheduling nodes can make packet routing and forwarding decisions without the notion of end-to-end routes. Moreover, they showed that combining the backpressure approach with WFQ and PRR approaches improves the efficiency of the energy cost with a small penalty in the delivery ratio. M. Alresaini et al. [28] also present a novel hybrid approach that they refer to as backpressure with adaptive redundancy (BWAR), which provides the best of both worlds.
This approach is highly robust and distributed and does not require any prior knowledge of network load conditions. They evaluated BWAR through both mathematical analysis and simulations based on cell-partitioned model. They prove theoretically that BWAR does not perform worse than traditional backpressure in terms of the maximum throughput, while yielding a better delay bound. T. Spyropoulos et al. [25] present a taxonomy of opportunistic routing protocols for DTNs, which serve as a set of guidelines for routing protocol designers and developers. They also take up some case studies of challenged wireless networks, and validate some of their routing design principles using simulations.

The performance of data routing heavily connects with the underlying network topology. Li et al. [30] study the topology design problem in a time-evolving and energy-harvesting WSN where the time-evolving topology and dynamic energy cost are known a priori or can be predicted, and model such a network as a node-weighted space-time graph. This kind of network is not delay tolerant. To reduce the cost of the network, they propose an efficient topology design problem to put more sensors into sleeping while still maintaining the network connectivity over time. It constructs a connected sparse sub graph from the original node-weighted space-time graph by removing nodes. Our goal is totally different. Instead of removing nodes, our approach tries to maintain a path with certain reliability between all pairs of nodes with the minimum cost by removing edges. Huang et al. [10] study topology control for predictable delay tolerant network. They model PDTN as a directed space-time graph, and define the topology control problem as finding a sub graph from this graph. They prove it is NP-complete. Two heuristic methods are proposed. Results of simulation demonstrate the efficiency of their proposed methods. However, their researches are based on deterministic connected network.

In practical wireless network, the links between nodes are probabilistic connected. To address this issue, the unreliable links are taken into consideration for topology control in PDTNs by Huang [24]. However, in the space-time directed graph they used, only one-hop connection is considered and the QoC is simplified as the probability of the worst link along the path.

3. System model

In PDTNs, all the devices are mobile, and the communication is not continuous among them. At certain time, some device can’t establish direct connection with each other. The devices move over time, so the topology of PDTNs changes dynamically with time (as shown in Fig.3), which means traditional static graphs cannot represent PDTNs well. To model such a kind of dynamically changed topology of PDTNs, we need to divide a certain time period into many time slots through discretization. At each time slot, a snapshot is taken at some point to represent the topology of this time slot. A series of snapshots can be combined together according to the time order to represent the topology of the whole network in a certain time period.

3.1. Three dimensional space-time weighted directed graph model

In Fig. 3, at each time slot, a static graph represents the snapshot topology. The nodes represent the mobile devices, and the edges represent the lossy links existing among the devices. The weight of each edge represents the connection probability of lossy link at this time slot.

The probability value can be obtained through estimating packet reception rate. First, each node sends a message with the minimum power, and we can get connection probability through packet reception rate. Then we gradually increase node’s transmission power. If the connection probability value obtained is very similar with the previous one before increasing power, we should stop increasing power and use the previous power as the power of the node. The goal is to get the maximum connection probability value with the minimum power. If a certain node sends information with maximum power and the connection probability value is still very small, such as 0.05, we can directly transform node into the listening state to save energy, because it is impossible for the node to communicate with other nodes in the moment. Executing the above process for each node at a certain time slot, a weighted directed graph of this time slot can be constructed. As shown in Fig.3 this graph is not completely connected.

The PDTN operate cyclically. The same topological change happens in every cycle. The weighted directed graphs representing the topology of certain time slots can be combined together according to the time order to model the PDTNs.

Assume that the cycle time is divided into some equal time slots, such as [1,···,T]. Let V=[V1,···,Vs] represent the set of nodes(wireless devices) in the network. Let E=[e1,···,em] be the set of all connection probability value of each edge for the corresponding directed graph at the t time slot. Let Gt = (Vt, Et, e') be a weighted directed graph representing the topology of the network at a time slot, and if edge VtVj exists, which indicates that there has a connection probability value bigger than setting value, and this connec-
The topology of PDTNs changes over time (a) a snapshot of the network, (b) time-varying topologies of the PDTNs.

Fig. 3. The topology of PDTNs changes over time (a) a snapshot of the network, (b) time-varying topologies of the PDTNs.

The connection probability value is made to be the weight of the edge $V_aV_b$. Therefore, dynamic network will be modeled as a union of a series of weighted directed graphs $\{G_t|t=1,\cdots,T\}$.

Let $G=(V, E, e)$ represents the three dimensional space-time weighted directed graph, including temporal and spatial information. Fig. 4 shows an example. There are $T$ time slots, so it means the model should include $T$ topological snapshots. At every time slot, there are $N$ nodes. The node set is represented by $V=\{V_j|j=1,\cdots,n, t=0,\cdots,T\}$. There are $n*T$ nodes in graph $G$. There are two types of edges, namely time and space connection. For the same node at different time slots, there exist time edges connecting them that are marked as red lines in the Fig.4. Because the same node at different time slots doesn’t need to deliver data but only inherit, we set the weight of time edge to 1.

The other edges are spatial edges, which come from the weighted directed graphs of different time slots.

To maximize the successful rate of data transfer between any two nodes, we can find the path with maximum connection probability from the three dimensional space-time weighted directed graph. For example, when the nodes $V_4$ and $V_6$ in Fig.4 try to communicate, a path $V_{41}V_{31}V_{32}V_{33}V_{63}V_{64}$ may be found to maximize connection probability. Because this path is a set of serial links, connection probability between $V_{41}$ and $V_{64}$ is the product of connection probability values of all the links along the path. The value is $0.9*1*1*0.9*1=0.81$, which means the node $V_4$ can transfer data to $V_6$ within 4 time slots at a successful rate 0.81.

### 3.2. Energy model

Three dimensional space-time weighted directed graph contains the information of network topology such as time, connection probability between nodes, and etc. From this graph, we can get the information about the chance that a node talks to another within a reasonable delay.

Due to the time discretization and adding the time edge, the complexity of model increases, which makes the routing decision more complicated. However, it also increases the solution space and leaves more opportunities for the link selecting and path establishing.

In PDTN, energy reserving is a very important factor to keep the network functional and the devices alive. The energy cost must be maintained in a reasonable level while trying to improve the QoC, especially when the devices are battery powered. An optimal tradeoff
between maximizing connection probability and minimizing the energy cost should be balanced.

In order to do that, the energy model for the nodes should be built first. The most significant source of energy consumption is the data transmission and it varies with the amount of data transferred and transmission power level. In our paper, we assume the nodes can adjust their transmission power level and the rest energy consumption besides data transmission is stable. Therefore, the links between the nodes have different energy consumption level.

In our model, the energy cost of the spatial edges at different time slots are given random value between 1 and 3. For the time edge (red edge) that represents the same node can carry the data to the next time slot waiting the opportunities to transfer to the other node, there is no actual data transmission, the energy cost is set to 0. Let $C(e)$ represents the energy cost of edge $e$, $C(G) = \sum C(e)$. where $e, e \in G$ and is used by the data transmission.

3.3. Problem statement

Generally, the aim of topology control is to find a substructure from the whole network to satisfy some certain performance demands with the lowest cost. In PDTNs, greedily using all the connection possibilities may improve the data delivery at a short time window at the cost of exhausting the energy of some nodes quickly and may degrade the performance of whole network after these nodes die.

Topology control based on probability proposed in this paper is different from deterministic topology control. Deterministic topology assumes the underlying links are deterministic. However, in practical environment, the link is not always connected deterministically, but probabilistically. Thus it is more practical for
PDTNs to use connection probability. Unlike data routing that finds the least cost path from the source node to destination, the method proposed in this paper focuses on maintaining a sub graph in which all the nodes are connected with certain reliability under certain cost.

In this paper we use one cycle time to represent the entire PDTNs, and we only consider the connection probability of each pair of nodes from the first slot to the last slot. If one message is received in the middle of one cycle, the message may not reach the destination in this cycle. However, in PDTNs the mobility pattern is predictable and cyclic, so the message can reach the destination in next cycle.

The graph model in this paper not only includes spatial and energy cost, but also includes temporal and connection probability information. The optimizing goals may be conflicted, for example, maximizing the connection probability and minimizing the energy cost. The topology control can be generalized as finding a sub graph from three dimensional space-time weighted directed graph to maximize or minimize a certain metric value (connection probability or energy cost) while satisfying certain constraints (connection probability or energy cost). This problem is a NP-Complete problem. Therefore, heuristic methods are used to solve it.

We now prove the NP-completeness of our topology control problem on three dimensional space-time weighted directed graph by using the theory of NP-Completeness that Salama, H.F. et al. [29] proposed. Their theory is as the following: Give a directed graph \( G=(V,E) \), \( m \) additive metrics \( d_1(a), d_2(a), \ldots, d_m(a) \) for each \( a \in E \), \( n \) multiplicative metrics \( d_1(a), d_2(a), \ldots, d_n(a) \) for each \( a \in E \), and two specified nodes \( i, d \), the problem of deciding if there is a simple path \( p=(i, j, k, \ldots, l, d) \) satisfying the above \( m+n(m+n+1)/2 \) metrics, in which any one metric is the objective and the other as the constraints is NP-complete. The definition of additive metric and multiplicative metric is as the following: Let \( d(i,j) \) be a metric for link \((i, j)\). For any path \( p=(i, j, k, \ldots, l, d) \), we say metric \( d \) is additive if 
\[
d(p) = d(i,j) + d(j,k) + \cdots + d(l,d),
\]
we say metric \( d \) is multiplicative if 
\[
d(p) = d(i,j) \times d(j,k) \times \cdots \times d(l,d).
\]

Theorem 1: Our newly defined topology control problem on three dimensional space-time weighted directed graph is a NP-complete problem.

proof: From above description of the NP-completeness theory, we can see our topology control problem can be reduced to above problem. Our topology control problem is to find a sub graph from three dimensional space-time weighted directed graph to maximize or minimize a certain metric value (connection probability or energy cost) while satisfying certain constraints (connection probability or energy cost). We can reduce our topology control problem to find the path of each pair of nodes of the directed graph that balances the connection probability and energy cost. Thus each pair of nodes is the same as the specified nodes \( i, d \). The energy cost of the path in our paper is calculated using the formula \( C(G) = \sum C(G) \), which is one additive metric, and the connection probability is calculated using \( ax \times bx \times \cdots \times c \) (\( a, b, \ldots, c \) are the connection probability of each link in the path), which is one multiplicative metric. Either connection probability or energy cost can be objective or constraint. In our model there may exist combined paths between each pair of nodes. In this case, it is easily to transform it to the multiplicative, such as using logarithmic formula approach on both sides. Therefore, the topology control problem on three dimensional space-time weighted directed graph is also NP-complete.

Since the problem is NP-Complete. To find a feasible solution to the practical applications, either the problem needs to be simplified or heuristic methods need to be developed. Different application scenarios may have different requirements on the sub structure built by topology control and focus on different goals. In this paper, two application scenarios are considered. One is to deliver some important data at a successful rate as high as it can reach. It is very useful to transfer data in the PDTNs that energy source is relatively powerful such as vehicle ad hoc network. The other is to deliver the data at a reasonable successful rate with lower cost. It fits the environment that energy is limited such as mobile sensor networks.

For previous one, the goal is to find a sub graph from the three dimensional space-time weighted directed graph maximizing connection probability of each pair of nodes. We simplify the problem to find a sub graph from the original model to make sure the connection probability between each pairs of nodes is maximized. It can be reduced to finding all pairs shortest path in a graph if excluding combined paths and only considering single paths. A heuristic algorithm with less complexity compared with Floyd’s algorithm is proposed.

For the last one, the aim is to construct a sub graph in which the connection probability of each pair of nodes is higher than a threshold to minimize the energy cost. Connectivity based on probability means that there is at least one single or combined path that satisfies a given threshold between each pair of nodes, so we can guarantee that any two nodes can complete the package delivery at a certain probability in a cycle time. Although some nodes are not connected at a certain time slot,
it is sure to find a path consisting of spatial and temporal edges between any two nodes based on three dimensional space-time weighted directed graph. If the threshold is appropriate, we can also find the paths that satisfy the conditions between any two nodes. Since it is NP-Complete, a heuristic algorithm is proposed to address this problem.

4. Topology control based on probability for DTNs

In this section, two heuristic topology control algorithms are proposed. The first algorithm is to find a sub graph to maximize the connection probability between each pair of nodes from the first time slot to the last time slot. The second one is to find a sub graph in which the connection probabilities between each pair of nodes are larger than the given threshold from the first time slot to the last time slot.

4.1. Maximize Connection Probability between each pair of nodes (MCP)

We propose a heuristic algorithm to find the paths between each pair of nodes with maximum connection probabilities. Firstly, we begin with the node Vi (i = 1, 2, · · ·, n) at the first time slot. Along the time link (red link in Fig. 4) of node Vi the corresponding node V (i) at the last time slot is found. Then we start from the last time slot to find the path with the maximum probability value connected to node j (j = 1, 2, · · ·, n) at the last time slot. If this path exists, it is saved. Afterward, along the time link back to the previous time slot, the path with the maximum probability value connected to node j (j = 1, 2, · · ·, n) at the current time slot is found. If it exists, we compare it with the previously stored value. And the stored one is replaced with the current one if the current one has a larger probability value. If the connection probability is the same, we compare the energy cost and the path with smaller energy cost is stored; otherwise we continue the above operation until it comes back to the first time slot. Finally, a path between node V (1) at the first time slot and node j at the last time slot can be established. After executing the above process n times, we can obtain the paths between node Vi at the first time slot and any node j (j = 1, 2, · · ·, n) at the last time slot. After performing the process for all the nodes, we find the paths between each pair of nodes from the first time slot to the last time slot, which forms a sub graph.

The pseudo code of above algorithm is as follows. \( n \) represents the number of nodes at each time slot and \( T \) represents the number of time slots, \( P_{\text{cur}} \) is the maximum connection probability calculated at current time slot, \( P_{\text{pre}} \) is the maximum probability calculated previously, \( E_{\text{cur}} \) is the energy cost of path calculated at current time slot, \( E_{\text{pre}} \) is the energy cost of path calculated previously.

Algorithm 1 Maximize Connection Probability between each pair of nodes (MCP).

**Input:**
- the original three dimensional space-time weighted directed graph, including connection probability and energy cost of each edge;

**Output:**
- the constructed sub graph, the ratio of total cost and the ratio of total edge number;

1: Obtain the paths between each pair of nodes at each time slot with largest connection probability according the Floyd algorithm;
2: for \( i = 1; i \leq n; i++ \) do
3: for \( j = 1; j \leq n; j++ \) do
4: for \( k = T - 2; k \geq 0; k-- \) do
5: if \( P_{\text{cur}} > P_{\text{pre}} \) then
6: \( P_{\text{pre}} = P_{\text{cur}} \);
7: else
8: if \( P_{\text{cur}} == P_{\text{pre}} \) then
9: if \( E_{\text{cur}} < E_{\text{pre}} \) then
10: \( E_{\text{pre}} = E_{\text{cur}} \);
11: end if
12: end if
13: end if
14: end for
15: end for
16: end for
17: Combine the paths into a sub graph. Calculate the ratio of total cost and the ratio of total edge number;

Line 1 to 17 is the core of the algorithm. Line 17 does estimating work. The time complexity of line 1 is \( O(n^3*T) \). Line 5 and 6 means if the maximum probability calculated at current time slot is larger than the previously stored one, the previous one is replaced. Line 7 to 10 means if the probability is the same, the energy cost will be compared, and the path with smaller energy cost will be stored. The time complexity of line 2 to 16 is \( O(n^2*T) \). The time complexity of line 17 is \( O(n^2*T) \). So the time complexity of the algorithm is \( O(n^3*T) \).

Theorem 2: The time complexity of the above algorithm is \( O(n^3*T) \), in which \( n \) represents the number of the nodes in the network, \( T \) represents the number of the time slots.
4.2. Satisfy the given Connection Probability Threshold (SCPT)

In this section we mainly describe the heuristic algorithm to find a sub graph under the condition of satisfying the probability threshold and reducing energy cost. Some basic theory about probability used in this paper is also explained.

4.2.1. Basic theory of probability

In this paper we mainly use the inclusion-exclusion principle of probability theory to calculate the connection probability between a pair of nodes connected by different topology, which may include the serial, parallel and serial-parallel topology as the previous example shown in Fig. 1. We give a simple example to illustrate how to calculate the probability of serial, parallel and serial-parallel topology in Fig. 5:

![Fig. 5. (a) Serial topology (b) Parallel topology (c) Serial-parallel topology](image)

Assume $V_s$ is the source node, $V_d$ is the destination node, and the probability of each pair of nodes is $\mu$. So the probability of serial topology shown in Fig. 5(a) is $\mu^2$, the probability of parallel topology shown in Fig. 5(b) is $1-(1-\mu^2)^2$, and the probability of serial-parallel topology shown in Fig. 5(c) is $(1-(1-\mu^2)^2)\mu$.

4.2.2. Main idea and pseudo code of the algorithm

We propose a heuristic algorithm to find the paths that satisfy the connection probability threshold. Firstly, we begin with the node $V_{ii}$ ($i=1,2,\cdots,n$) at the first time slot, and along the time link (red link in Fig. 4) of node $V_{ii}$ we find the corresponding node $V_{ij}$ at the last time slot. Then we start from the last time slot to find the path with the maximum probability connected to node $j$ ($j=1,2,\cdots,n$) at the last time slot. If it exists, we will calculate the energy cost $C(L)$ according to energy model, and we save the path information and $C(L_k)$. Afterwards, along the time edge back to the previous time slot, the path with the maximum probability connected to node $j$ ($j=1,2,\cdots,n$) is found at the current time slot. If it exists, the same work described as the above process will be done, such as storing the path information and $C(L_{k+1})$. The above operation will be continued until getting back to the first time slot. Some paths between node $V_{ii}$ at the first time slot and node $j$ at the last time slot with larger probability will be obtained. The probabilities of all obtained paths are compared with the given threshold.

If some of these obtained paths have the probability values larger than or equal to the given threshold, the path with the minimal energy cost will be chosen as the path between node $V_{ii}$ at the first time slot and node $j$ ($j=1,2,\cdots,n$) at the last time slot. If none of these paths has the probability value larger than or equal to the given threshold, the path with the maximum probability value will be selected firstly, and then the second largest one will be chosen too. The probability value of the combined paths of these two paths can be calculated according to the inclusion-exclusion principle of probability theory. If the probability value is larger than the given threshold, the combined path between these two nodes will be stored. Otherwise, the thirdly largest one will be combined using the same method above, and so on, until finding a combined path that satisfies the given threshold. Of course, if the given threshold is unreasonable, it is possible that there will be no path satisfying the given threshold.

After executing the above process $n$ times, the path or combined path that satisfies the given threshold between node $V_{ii}$ at the first time slot and any node $j$ ($j=1,2,\cdots,n$) at the last time slot is obtained. After $n$ cycles, the paths or combined paths that satisfy the given threshold between each pair of nodes from the first time slot to the last time slot will be found. Thus a sub graph is constructed.

The pseudo code of above algorithm is as follows, bh represent whether or not there is a single path whose maximum probability value is larger than the threshold, $r$ is the given threshold.

Line 1 to 26 is the core of the algorithm. Line 27 does calculation work. The time complexity of line 1 is $O(n^3*T)$. The time complexity of line 2 is $O(n^2*T*\log T)$. Line 6 to 10 is to estimate whether the maximum connection probability obtained at each time slot is larger than the threshold. If it is true, the id of the time slot is recorded. Line 11 is used to estimate whether there exist a path whose probability value is larger or equal to the threshold. If it exists, line 12 will be executed. If the probability values of all the paths are smaller than the threshold, multiple paths will be required to combine together to satisfy the threshold, line 14 to 26 will be executed. The time complexity of line 3 to 26 is $O(n^2*T^2)$, the time complexity of line 27 is $O(n^2*\log T)$. So the time complexity of the algorithm is $O(n^3*T+n^2*T^2)$.

Theorem 3: The time complexity of the above algorithm is $O(n^3*T+n^2*T^2)$, in which $n$ represents the num-
Algorithm 2 Satisfy the given Connection Probability Threshold (SCPT).

**Input:**
the original three dimensional space-time weighted directed graph, including connection probability and energy cost of each edge;

**Output:**
the constructed sub graph, the ratio of total cost and the ratio of total edge number ;

1. Obtain the paths between each pair of nodes at each time slot with largest connection probability according the Floyd algorithm;
2. Sort connection between each pair of nodes at every time slot in ascending order according to the connection probability;
3. for \(i = 1; i \leq n; i + + \) do
4. for \(j = 1; j \leq n; j + + \) do
5. \(bh = -1;\)
6. for \(k = T - 1; k \geq 0; k - - \) do
7. if \(P_{cur} \geq \tau\) then
8. \(bh = k;\)
9. end if
10. end for
11. if \(bh > -1\) then
12. choose the path with the minimum energy cost from the paths with the probability larger than the threshold;
13. else
14. \(P_{cur}=1-P_{cur}[T-1];\) //The maximum connection probability is assigned to the current value
15. for \(m = T - 1; m \geq 0; m - - \) do
16. \(P_{cur}^*=(1-P_{cur}[m]);\) //The path with the second largest probability is combined with current one and new P_{cur} is calculated
17. if \(P_{cur}<(1-\tau)\) then
18. for \(n = 0; m \leq T; n + + \) do
19. \(S[i][j][n]=P_{cur}[m];\) // \(S[i][j][n]\) is used to store the paths between node \(i\) and \(j\)
20. end for
21. break;
22. end if
23. end for
24. end if
25. end for
26. end for
27. Combine the paths into a sub graph. Calculate the ratio of total cost and the ratio of total edge number;

5. Performance evaluation and analysis

In this section, MCP and SCPT will be evaluated through simulations. To the best of our knowledge, GDL [24] is the latest algorithm proposed to address the topology control of DTN with lossy link. We compared SCPT with it.

5.1. Performance metrics and parameters

In simulation, we mainly consider the following performance metrics and parameters:

(1) The ratio of total cost: the total cost of the constructed sub graph in term of energy consumption. We use the ratio against the cost of the original network to show how much saving achieved by the topology control algorithm, comparing to the original network without topology control.

(2) The ratio of total edge number: the number of edges in the constructed sub graph. We use the ratio against the number of edges in the original network to show how much saving achieved by the topology control algorithm.

(3) Link density \(\rho\): It is equal to dividing the number of edges in the current topology by the number of total edges in the complete connected topology. It is used to control the topology of PDTN in the simulation. The small \(\rho\) leads to a sparse PDTN. \(\rho=1.0\) implies that the topology at each time slot is a complete connected graph.

(4) Threshold \(\tau\): It is the value preset according to the need of the practical application.

In our simulation, we assume that PDTN have 11 nodes and spread over 9 time slots. For space edge, the connection probability is given a random value which is larger than 0 and smaller than 1 and the energy cost is randomly chosen from 1 to 3. For time edge, the connection probability is 1 and the energy cost is 0. For all the simulations, we repeat the experiment for five times and take the average value.

5.2. Results analysis

5.2.1. Maximize Connection Probability between each pair of nodes (MCP)

In this section, we evaluate our MCP algorithm. To demonstrate the performance of our algorithm, we also use Floyd’s algorithm to find the paths with largest connection probabilities from the original model, which
can get the optimal single path with the maximum connection probability. We call it as OPT algorithm. The average connection probability, the ratio of total cost and the ratio of total edge number are compared with different original network topology generated by different link density $\rho$.

Fig. 6 describes the relationship between the average connection probability and the link density $\rho$. As shown in the figure, the average connection probability increases while $\rho$ increases and the value is close to 1 if $\rho$ is larger. This is because there are more probabilistic links existing in the original network topology with higher $\rho$ value. And in the PDTNs the existing of time edges can improve the successful data delivery significantly because we count their probability as 1 in one cycle.

The results of MCP algorithm are very close to OPT. Besides the case of $\rho = 0.1$, the difference between MCP and OPT is only 5%. When $\rho$ is very small, MCP, as a heuristic method, may not find the best path. However, the complexity of OPT is $O(n^3T^3)$, which is higher than MCP with $O(n^3T)$. It indicates that MCP is efficient and stable, which can find the path with the connection probability close to the optimal value of single path.

Fig. 7 describes the relations between the ratio of total cost and link density $\rho$. As shown in the figure, the ratio of total cost decreases while $\rho$ increases. MCP is always lower than OPT. When $\rho$ is 0.9, the ratio of total edge number drops to around 0.092. Even when $\rho$ is 0.1, the ratio of total edge number is around 0.69. It indicates that MCP can use less edges to reach a higher connection probability, which makes the topology of whole network more sparse and easier to maintain, and reduces the mutual interference between the nodes.

Fig. 8 describes the relations between the ratio of total edge number and link density $\rho$. Similar with the energy cost, the ratio of total edge number decreases while $\rho$ increases. MCP is always lower than OPT. When $\rho$ is 0.9, the ratio of total edge number drops to around 0.092. Even when $\rho$ is 0.1, the ratio of total edge number is around 0.69. It indicates that MCP can use less edges to reach a higher connection probability, which makes the topology of whole network more sparse and easier to maintain, and reduces the mutual interference between the nodes.
5.2.2. Satisfy the given Connection Probability Threshold (SCPT)

In this section, we will compare SCPT with GDL in terms of the ratio of total cost and the ratio of total edge number with different threshold $\tau$ and link density $\rho$.

Fig. 9 describes the relations between the ratio of total cost and the threshold $\tau$ with certain link density $\rho$. $\tau$ varies from 0.6 to 0.9, and $\rho$ is set to 0.3 and 0.7. The reason that threshold $\tau$ starts from 0.6 is that in most cases the network can transmit data efficiently in a cycle only when the connection probability is not very small.

As shown in the figure, the ratio of total cost increases with $\tau$ increasing no matter what algorithm is adopted. Our algorithm is always better than GDL. And when $\tau$ is higher than 0.8 with $\rho=0.7$, GDL cannot find a sub graph satisfying the threshold. In this case, SCPT still can construct a sub graph satisfying the threshold even $\tau$ reaches 0.9. It is because GDL only considers single hop link at each time slot. The paths between each pair of nodes found by GDL are serial. Our algorithm considers multi-hop link at each time slot. Multiple single paths are also combined together to improve connection probability. Compared with the topology structure generated by GDL in which the connection probability is only a little bigger or equal than the threshold $\tau$, the connection probability between each pair of nodes in our topology is higher.

It also can be found in the Fig. 9 that higher $\rho$ value leads to larger saving in energy cost. When $\rho$ is 0.7, the ratio of total cost varies from 0.13 to 0.34 with different threshold $\tau$. In the worst case it can save 66% energy cost. When $\rho$ is 0.3, the ratio of total cost is in the range of 0.26-0.45, which indicates that our algorithm can save about 55% energy cost. We can identify that even in the sparse network, especially in the challenging environment discussed in this paper, SCPT also can save energy.

The relations between the ratio of total edge number and the threshold $\tau$ are described in Fig. 10. $\tau$ changes from 0.6 to 0.9, and the link density $\rho$ is set to 0.3 and 0.7. As shown in Fig. 10, the ratio of total edge number of our algorithm SCPT increases slowly when $\tau$ increases. The ratio of total edge number of GDL increases faster with $\tau$ increasing. When $\tau$ is bigger than 0.75 with $\rho=0.3$, GDL cannot find sub graph that satisfies the threshold with the same reason we discussed before.

Our algorithm is always better than GDL under the same link density $\rho$. The reason is that GDL only takes one hop connection into consideration at each time slot. The changing trend of the ratio of total edge number of our algorithm is similar with Fig. 9. When the $\tau$ is bigger, it uses a lot of links trying to get a high connection probability. It is because the total number of edges has direct relation with the total energy cost, the more edges it has, and the more energy it costs. The ratio of total edge numbers changes between 0.12 and 0.23 when $\rho$ is 0.7. In the worst case it only uses 77% edges from the original topology. It also indicates that our algorithm can exclude more edges from the original topology with the increasing of $\rho$, and the formed network is very sparse. Even if when $\rho$ is 0.3, the ratio of total edge number is from 0.3 to 0.45, and our algorithm can save about 55% edges in the worst case, which indicates even in the sparse network environment our algorithm can construct a sparse topology that satisfies a
The high threshold value.

Fig. 11 describes the relations between the ratio of total cost and the link density $\rho$ with certain threshold $\tau$. $\rho$ changes from 0.1 to 0.7, and $\tau$ is set to 0.6 and 0.8. The ratio of total cost decreases when $\rho$ increases. Higher $\rho$ value means there are more links existing in the original topology. Topology control can achieve more performance gains.

Our algorithm is always better than GDL. When $\rho$ is smaller than 0.4 with $\tau=0.6$, or $\rho$ is smaller than 0.3 with $\tau=0.8$, GDL cannot find a sub graph that satisfies the corresponding threshold. SCPT can always find a sub graph under these circumstances, even when $\rho$ is 0.1. The reason is similar, multiple hop connection at one time slot and multiple paths are considered.

With higher $\tau$, the ratio of total cost of GDL increases faster since more edges are needed to get a higher connection probability. It indicates that GDL performs worse when $\tau$ is larger. The ratio of the range of total cost of SCPT is between 0.13 and 0.39 when $\tau$ is 0.6, and even in the worst case it can save 63% energy cost. When $\tau$ is 0.8, the ratio of total cost is in the range of 0.21-0.52, which indicated that even under the circumstance of high threshold $\tau$, our algorithm can also save a significant amount of energy. Thus our algorithm is more effective when the application requires higher connection probability.

Fig. 12 describes the relations between the ratio of total edge number and the link density $\rho$. The ratio of total edge number decreases when $\rho$ increases. Our algorithm is always better than GDL. The ratio of total edge number of GDL increases with $\tau$ increasing. When $\tau$ reaches a high value and $\rho$ is smaller than 0.3, GDL cannot find the solution, which also indicated that GDL performs worse when $\tau$ is larger and $\rho$ is smaller. The tendency of our algorithm is similar with Fig. 11. The ratio of total edge number drop to the lowest value about 0.1 when $\rho$ is 0.7. That again indicates that our algorithm can construct a sparse topology network.

6. Conclusion

In this paper, we have studied topology control based on probability for PDTNs. We define this problem as finding a sub graph from three dimensional space-time weighted directed graph to balance the energy cost and connection probability. Two heuristic topology control algorithms MCP and SCPT are proposed. Extensive simulation experiments demonstrate that our methods can achieve higher connection probability with lower energy cost.

Future works include designing more effective algorithms, especially distributed algorithms and extending our methods to random DTNs.

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References


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